Degree and Order of Differential Equation

The highest order derivative of any equation is called order of differential equation and Power of highest order derivative is called degree of equation.

Example $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^4 + y = 2$

highest order derivative is 2 and its Power is also 2

$\Rightarrow$ Order $= 2$ Degree $= 2$

Questions of Linear Differential Equation

$\frac{dy}{dx} + Py = Q$, where P and Q are function of x only.

or

$\frac{dx}{dy} + Px = Q$ where P and Q are only Function of y.

Solution: Find Integrating Factor $e^{\int P dx}$.

then Solution is $y \cdot e^{\int P dx} = \int Q e^{\int P dx} + C$

1. $\frac{dy}{dx} + y\tan x = \sec x$

I.F $e^{\int \tan x \, dx} = e^{\int \left(\frac{\sin x}{\cos x}\right) dx} = e^{-\int -\frac{\sin x}{\cos x} dx}$

I.f $e^{-\log \cos x} = e^{-\log \cos x} = e^{\log \cos x^{-1}} = \frac{1}{\cos x}$

I.f $\sec x$

Solution is $y.\sec x = \int \sec x . \sec^2 x \, dx = c$

$\frac{y}{\cos x} = \int \sec^2 x \, dx + c$

$\frac{y}{\cos x} = \tan x + c$

$y = \cos x \left(\frac{\sin x}{\cos x} \right) + c \cos x$

$\Rightarrow y = \sin x + c \cos x$

2. $\frac{dy}{dx} = \frac{x+y+1}{x+1}$

Solution:-- $\frac{dy}{dx} = \frac{(x+1)}{(x+1)} + \frac{y}{(x+1)}$

$\frac{dy}{dx} = 1 + \frac{y}{x+1}$

$\frac{dy}{dx} - \frac{y}{x+1} = 1$

first Convert the equation in $\frac{dy}{dx} + Py = Q$ form

I.f $e^{-\int \frac{1}{x+1} dx}$

$= e^{-\log(x+1)} = e^{\log(x+1)^{-1}}$

I.f $= \frac{1}{x+1}$

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Solution is

$$y \cdot \frac{1}{x+1} = \int \frac{1}{x+1} \cdot 1 \, dx + c$$

$$\frac{y}{x+1} = \log(x+1) + c$$

3. $\frac{dy}{dx} + \frac{2x}{x^2+1} \cdot y = \frac{1}{(x^2+1)^2}$ given that $y = 0$ when $x = 1$

$$\text{I.F } e^{\int \frac{2x}{x^2+1} dx} = e^{\int \frac{d(x^2+1)}{x^2+1} dx}$$

$$\Rightarrow \text{I.F } e^{\log(x^2+1)} = x^2+1$$

Solution is

$$y \cdot (x^2+1) = \int x^2+1 \cdot \frac{1}{(x^2+1)^2} dx + c$$

$$y \cdot (x^2+1) = \int \frac{1}{x^2+1} dx + c$$

$$y \cdot (x^2+1) = \tan^{-1} x + c$$

Put $x=1$ $y=0$

$$0 \cdot (2) = \tan^{-1} + c$$

$$0 = \frac{\pi}{4} + c$$

$$\Rightarrow c = -\pi/4$$

$$\Rightarrow y \cdot (x^2+1) = \tan^{-1} x - \pi/4$$

4. $x(x-1)\frac{dy}{dx} - y = x^2(x-1)^2$

$$\frac{dy}{dx} - \frac{y}{x(x-1)} = \frac{x^2(x-1)^2}{x(x-1)}$$

$$\frac{dy}{dx} - \frac{y}{x(x-1)} = x(x-1)$$

I.f $e^{-\int \frac{1}{x(x-1)} dx}$

put $\frac{1}{x(x-1)} = \frac{1}{x} - \frac{1}{x-1}$

I.f $e^{\int \left(\frac{1}{x} - \frac{1}{x-1}\right) dx}$

$= e^{\log x - \log x-1}$

$= e^{\log \frac{x}{x-1}}$

$\Rightarrow$ I.f $= \frac{x}{x-1}$

Solotion is

$$y \cdot \frac{x}{x-1} = \int \frac{x}{\cancel{x-1}} \cdot x(\cancel{x-1})\, dx + c$$

$$y \cdot \frac{x}{x-1} = \int x^2\, dx + c$$

$$y \cdot \frac{x}{x-1} = \frac{x^3}{3} + c$$

$$y = \left(\frac{x-1}{x}\right)\left(\frac{x^3}{3} + c\right)$$

$$= \left(1 - \frac{1}{x}\right)\left(\frac{x^3}{3} + c\right)$$

5. $(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$

Change given equation in $\frac{dx}{dy} + Px = Q$

It is linear equation in x

$$\frac{dx}{dy} = \tan^{-1}y - x \,/\, 1+y^2$$

$$\frac{dx}{dy} = \frac{\tan^{-1}y}{1+y^2} - \frac{x}{1+y^2}$$

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$$

$$I.F \quad e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$$

Solution is

$$x.\,e^{\tan^{-1}y} = \int e^{\tan^{-1}y}\,\frac{\tan^{-1}y}{1+y^2}\,dy + c$$

Put $\tan^{-1}y = z$ $\quad \frac{1}{1+y^2}dy = dz$

$$x.e^{\tan^{-1}y} = \int e^z.z\,dz + c$$

$$x.e^z = ze^z - e^z + c$$

$$x = (z-1) + ce^{-z}$$

$$x = (\tan^{-1}y - 1) + c\,e^{-\tan^{-1}y}$$

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Linear Differential Equations of the First Order

2.1 DEFINITION

A differential equation is said to be linear if the dependent variable and its derivative occur only in the first degree and are not multiplied together.

Thus, the standard form of a linear differential equation of the first order is $\frac{dy}{dx} + \text{P}y = \text{Q}$, where P and Q are functions of x or constants (*i.e.*, independent of y).

2.2 TO SOLVE THE EQUATION $\frac{dy}{dx} + \text{P}y = \text{Q}$, WHERE P AND Q ARE FUNCTIONS OF x ONLY (Leibnitz's Equation)

The given equation is $\frac{dy}{dx} + \text{P}y = \text{Q}$

Multiplying throughout by $e^{\int \text{P}\,dx}$, we get

$$\frac{dy}{dx} \cdot e^{\int \text{P}\,dx} + \text{P}y \cdot e^{\int \text{P}\,dx} = \text{Q} \cdot e^{\int \text{P}\,dx} \qquad \text{...(1)}$$

Now,
$$\frac{d}{dx}\left[ye^{\int \text{P}\,dx}\right] = \frac{dy}{dx} \cdot e^{\int \text{P}\,dx} + y \cdot \frac{d}{dx}\left[e^{\int \text{P}\,dx}\right]$$

$$= \frac{dy}{dx} \cdot e^{\int \text{P}\,dx} + y \cdot e^{\int \text{P}\,dx} \cdot \frac{d}{dx}\left[\int \text{P}\,dx\right]$$

$$\left[\because \frac{d}{dx}\left\{e^{f(x)}\right\} = e^{f(x)} \cdot \frac{d}{dx}\{f(x)\}\right]$$

$$= \frac{dy}{dx} \cdot e^{\int \text{P}\,dx} + y \cdot e^{\int \text{P}\,dx} \cdot \text{P} = \frac{dy}{dx} \cdot e^{\int \text{P}\,dx} + \text{P}y \cdot e^{\int \text{P}\,dx}$$

$\therefore$ From (1), $\frac{d}{dx}\left[y \cdot e^{\int \text{P}\,dx}\right] = \text{Q} \cdot e^{\int \text{P}\,dx}$

Integrating both sides w.r.t. x, we have

$$y \cdot e^{\int \text{P}\,dx} = \int \text{Q} \cdot e^{\int \text{P}\,dx}\,dx + c$$

which is the required solution of the given linear differential equation.

Note 1. The factor $e^{\int P\,dx}$, on multiplying by which the LHS of the differential equation becomes the differential co-efficient of some function of x and y, is called an integrating factor of the differential equation and is shortly written as I.F.

Note 2. The solution of the linear equation $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x only, is

$$y(\text{I.F.}) = \int Q(\text{I.F.})\,dx + c$$

Note 3. Sometimes a differential equation becomes linear if we take y as the independent variable and x as dependent variable. In that case, the equation can be put in the form $\frac{dx}{dy} + Px = Q$, where P and Q are functions of y (and not of x) or constants.

I.F. (in this case) $= e^{\int P\,dy}$, and the solution is

$$x(\text{I.F.}) = \int Q.\,(\text{I.F.})\,dy + c.$$

Note 4. While evaluating the I.F., it is very useful to remember that $e^{\log f(x)} = f(x)$.

Thus, $$e^{\log x^2} = x^2.$$

Note 5. The co-efficient of $\frac{dy}{dx}$, if not unity, must be made unity by dividing throughout by it.

ILLUSTRATIVE EXAMPLES

Example 1. *Solve the following :*

(*i*) $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$ (*ii*) $\frac{dy}{dx} = y\tan x - 2\sin x.$

Sol. (*i*) Given equation is $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$

Dividing throughout by $1+x^2$, $\left(\text{to make the co-efficient of } \frac{dy}{dx} \text{ unity.}\right)$

$$\frac{dy}{dx} + \frac{2x}{1+x^2}\cdot y = \frac{4x^2}{1+x^2} \quad ...(i)$$

It is of the form $\frac{dy}{dx} + Py = Q$

Here, $$P = \frac{2x}{1+x^2},\ Q = \frac{4x^2}{1+x^2}$$

$\therefore$ $$\text{I.F.} = e^{\int P\,dx} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = 1+x^2$$

Hence the solution is

$$y\,.\,(\text{I.F.}) = \int Q\,.\,(\text{I.F.})\,dx + c$$

or $$y(1+x^2) = \int \frac{4x^2}{1+x^2}\,.\,(1+x^2)\,dx + c$$

or $$y(1+x^2) = \int 4x^2 dx + c$$

or $$y(1+x^2) = \frac{4x^3}{3} + c.$$

(*ii*) Given equation is $\dfrac{dy}{dx} - (\tan x) \cdot y = -2\sin x$

It is of the form $\dfrac{dy}{dx} + Py = Q$

Here $$P = -\tan x,\ Q = -2\sin x$$

$\therefore$ $$\text{I.F.} = e^{\int P\,dx} = e^{-\int \tan x\,dx} = e^{-(-\log \cos x)}$$
$$= e^{\log \cos x} = \cos x$$

Hence the solution is

$$y\,(\text{I.F.}) = \int Q\,.\,(\text{I.F.})\,dx + c$$

or $$y\cos x = \int -2\sin x \cos x\,dx + c$$
$$= -\int \sin 2x\,dx + c = -\frac{-\cos 2x}{2} + c$$

or $$y\cos x = \tfrac{1}{2}\cos 2x + c.$$

Example 2. *Solve the following*:

(*i*) *sec x* $\dfrac{dy}{dx} = y + \sin x$ (*ii*) $x \log x \dfrac{dy}{dx} + y = 2\log x$

Sol. (*i*) Given equation is $\sec x \cdot \dfrac{dy}{dx} - y = \sin x$

Dividing throughout by sec x, to make the co-efficient of $\dfrac{dy}{dx}$ unity,

$$\frac{dy}{dx} - (\cos x)\,.\,y = \sin x \cos x$$

It is of the form $$\frac{dy}{dx} + Py = Q$$

Here, $$P = -\cos x, \quad Q = \sin x \cos x$$

$\therefore$ $$\text{I.F.} = e^{\int P\,dx} = e^{\int -\cos x\,dx} = e^{-\sin x}$$

Hence the solution is

$$y\,.\,(\text{I.F.}) = \int Q\,.\,(\text{I.F.})\,dx + c$$

or $$y\,.\,e^{-\sin x} = \int \sin x \cos x\,.\,e^{-\sin x}\,dx + c = \int te^{-t}\,dt + c, \text{ where } t = \sin x$$
$$= t\,.\,\frac{e^{-t}}{-1} - \int 1.\frac{e^{-t}}{-1}\,dt + c = -te^{-t} - e^{-t} + c$$
$$= -e^{-t}(t+1) + c = -e^{-\sin x}(\sin x + 1) + c$$

or $$y = -(\sin x + 1) + c\,e^{\sin x}.$$

(*ii*) Given equation is $x \log x \frac{dy}{dx} + y = 2 \log x$

Dividing throughout by $x \log x$ to make the co-efficient of $\frac{dy}{dx}$ unity,

$$\frac{dy}{dx} + \frac{1}{x \log x} \cdot y = \frac{2}{x}.$$

It is of the form $\frac{dy}{dx} + \text{P}y = \text{Q}$

Here, $\text{P} = \frac{1}{x \log x}, \quad \text{Q} = \frac{2}{x}$

$\therefore$ $$\text{I.F.} = e^{\int \text{P}\, dx} = e^{\int \frac{1}{x \log x} dx} = e^{\int \frac{1/x}{\log x} dx} = e^{\log \log x} = \log x$$

Hence the solution is

$$y \,.\, (\text{I.F.}) = \int \text{Q}.\,(\text{I.F.})\, dx + c$$

or $$y \log x = \int \frac{2}{x} \log x \, dx + c$$

or $$y \log x = 2 \int \frac{1}{x} \,.\, \log x \, dx + c = 2 \,.\, \frac{(\log x)^2}{2} + c$$

$\because \int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1}, n \neq -1$

or $$y \log x = (\log x)^2 + c.$$

Example 3. *Solve:* $x(x-1) \frac{dy}{dx} - (x-2)y = x^3(2x-1).$

Sol. Given equation is

$$x(x-1) \frac{dy}{dx} - (x-2)y = x^3(2x-1)$$

or $$\frac{dy}{dx} - \frac{x-2}{x(x-1)} y = \frac{x^2(2x-1)}{x-1}$$

It is of the form $\frac{dy}{dx} + \text{P}y = \text{Q}$

Here, $\text{P} = -\frac{x-2}{x(x-1)}, \quad \text{Q} = \frac{x^2(2x-1)}{x-1}$

$\therefore$ $$\text{I.F.} = e^{\int \text{P}\, dx} = e^{-\int \frac{x-2}{x(x-1)} dx} = e^{-\int \left(\frac{2}{x} - \frac{1}{x-1}\right) dx}$$

$$= e^{-[2 \log x - \log (x-1)]} = e^{-[\log x^2 - \log (x-1)]}$$

$$= e^{-\log \frac{x^2}{x-1}} = e^{\log \left(\frac{x^2}{x-1}\right)^{-1}} = \left(\frac{x^2}{x-1}\right)^{-1} = \frac{x-1}{x^2}$$

$\therefore$ The solution is

$$y \cdot \frac{x-1}{x^2} = \int \frac{x^2(2x-1)}{x-1} \cdot \frac{x-1}{x^2}\, dx + c = \int (2x-1)\, dx + c = x^2 - x + c$$

or $$y(x-1) = x^2(x^2 - x + c).$$

Example 4. *Solve:* $x(1-x^2)\dfrac{dy}{dx} + (2x^2-1)y = x^3.$

Sol. Dividing by $x(1-x^2)$ to make the co-efficient of $\dfrac{dy}{dx}$ unity, the given equation becomes

$$\frac{dy}{dx} + \frac{2x^2-1}{x(1-x^2)}\, y = \frac{x^2}{1-x^2}$$

It is of the form $\dfrac{dy}{dx} + Py = Q$

Here $$P = \frac{2x^2-1}{x(1-x^2)}, \quad Q = \frac{x^2}{1-x^2}$$

Now $$P = \frac{2x^2-1}{x(1-x)(1+x)} = -\frac{1}{x} + \frac{1}{2(1-x)} - \frac{1}{2(1+x)}$$

[Partial fractions]

$$\therefore \quad \int P dx = -\log x - \frac{1}{2}\log(1-x) - \frac{1}{2}\log(1+x)$$

$$= -\log[x(1-x)^{1/2}(1+x)^{1/2}]$$

$$= -\log[x\sqrt{1-x^2}] = \log(x\sqrt{1-x^2})^{-1}$$

$$\therefore \quad \text{I.F.} = e^{\int P\, dx} = e^{\log(x\sqrt{1-x^2})^{-1}} = \frac{1}{x\sqrt{1-x^2}}$$

The solution is

$$y \cdot \frac{1}{x\sqrt{1-x^2}} = \int \frac{x^2}{1-x^2} \cdot \frac{1}{x\sqrt{1-x^2}}\, dx + c$$

$$= \int \frac{x}{(1-x^2)^{3/2}}\, dx + c = -\frac{1}{2}\int (1-x^2)^{-3/2} \cdot (-2x)\, dx + c$$

$$= -\frac{1}{2} \cdot \frac{(1-x^2)^{-1/2}}{-\frac{1}{2}} + c$$

$$\Rightarrow \quad \frac{y}{x\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}} + c \quad \Rightarrow \quad y = x + cx\sqrt{1-x^2}$$

which is the required solution.

Example 5. *Solve:* $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1.$

Sol. The given equation is

$$\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$$

or $$\frac{dy}{dx} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}} \quad \text{or} \quad \frac{dy}{dx} + \frac{1}{\sqrt{x}}y = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

It is of the form $$\frac{dy}{dx} + Py = Q$$

Here $$P = \frac{1}{\sqrt{x}}, \quad Q = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

$\therefore$ $$\text{I.F.} = e^{\int \frac{1}{\sqrt{x}}dx} = e^{\int x^{-1/2}\,dx} = e^{2\sqrt{x}}$$

$\therefore$ Hence the solution is

$$y \,.\, e^{2\sqrt{x}} = \int \frac{e^{-2\sqrt{x}}}{\sqrt{x}} \,.\, e^{2\sqrt{x}}\,dx + c$$

or $$y \,.\, e^{2\sqrt{x}} = \int \frac{1}{\sqrt{x}}\,dx + c$$

or $$ye^{2\sqrt{x}} = 2\sqrt{x} + c \quad \text{or} \quad y = e^{-2\sqrt{x}}(2\sqrt{x} + c).$$

Equations of the Form $\frac{dx}{dy}$ + Px = Q where P and Q are functions of y only.

Example 6. *Solve the following:*

(*i*) $(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$ (*ii*) $(2x - 10y^3)\frac{dy}{dx} + y = 0.$

Sol. (*i*) The given equation is

$$(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$$

or $$(1 + y^2)\frac{dx}{dy} + x - e^{\tan^{-1} y} = 0$$

or $$\frac{dx}{dy} + \frac{1}{1+y^2}\,.\,x = \frac{e^{\tan^{-1} y}}{1+y^2}$$

It is of the form $$\frac{dx}{dy} + Px = Q$$

$$\text{I.F.} = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1} y}$$

$\therefore$ The solution is

$$x \cdot e^{\tan^{-1} y} = \int \frac{e^{\tan^{-1} y}}{1+y^2} \cdot e^{\tan^{-1} y}\, dy + c = \int e^t \cdot e^t\, dt + c \quad \text{where } t = \tan^{-1} y$$

$$= \int e^{2t}\, dt + c = \tfrac{1}{2} e^{2t} + c$$

or
$$x \cdot e^{\tan^{-1} y} = \tfrac{1}{2} e^{2\tan^{-1} y} + c.$$

(*ii*) The given equation is $(2x - 10y^3)\dfrac{dy}{dx} + y = 0$

or
$$y \cdot \frac{dx}{dy} + 2x - 10y^3 = 0 \quad \text{or} \quad \frac{dx}{dy} + \frac{2}{y} \cdot x = 10y^2$$

It is of the form $\dfrac{dx}{dy} + \text{P}x = \text{Q}$

$$\text{I.F.} = e^{\int \text{P}\, dx} = e^{\int \frac{2}{y}\, dy} = e^{2 \log y} = e^{\log y^2} = y^2$$

$\therefore$ The solution is
$$xy^2 = \int 10y^2 \cdot y^2 dy + c = 10 \int y^4\, dy + c$$

or
$$xy^2 = \frac{10y^5}{5} + c = 2y^5 + c.$$

TEST YOUR KNOWLEDGE

Solve the following differential equations:

1. $\dfrac{dy}{dx} + \dfrac{y}{x} = x^2$
2. $\dfrac{dy}{dx} + y \sec x = \tan x$
3. $\dfrac{dy}{dx} + y \tan x = \sec x$
4. $(1 + x^2)\dfrac{dy}{dx} + 2xy = \cos x$
5. $\dfrac{dy}{dx} = \dfrac{x + y + 1}{x + 1}$
6. $(x + 1)\dfrac{dy}{dx} - ny = e^x (x + 1)^{n+1}$
7. $\cos^2 x \dfrac{dy}{dx} + y = \tan x$
8. $(1 + x^2)\dfrac{dy}{dx} + y = \tan^{-1} x$
9. $\dfrac{dy}{dx} + \dfrac{2x}{x^2 + 1} \cdot y = \dfrac{1}{(x^2+1)^2}$ given that $y = 0$ when $x = 1$
10. $\dfrac{dy}{dx} + 2y \tan x = \sin x$ given that $y = 0$ when $x = \dfrac{\pi}{3}$
11. $x\dfrac{dy}{dx} + 2y = x^2 \log x$
12. $\dfrac{dy}{dx} + y \cos x = \sin 2x$
13. $\dfrac{dy}{dx} = x(x^2 - 2y)$
14. $\sin x \dfrac{dy}{dx} + y \cos x = 2 \sin^2 x \cos x$
15. $(1 - x^2)\dfrac{dy}{dx} + xy = ax$
16. $x(x - 1)\dfrac{dy}{dx} - y = x^2(x - 1)^2$

17. $y\,dx - x\,dy + \log x\,dx = 0$

18. $\dfrac{dy}{dx} + 2y \cot x = 3x^2 \operatorname{cosec}^2 x$

19. $\sin 2x \dfrac{dy}{dx} = y + \tan x$

20. $(x + 2y^3) \dfrac{dy}{dx} = y$

21. $(1 + y^2)\,dx = (\tan^{-1} y - x)\,dy$

22. $e^y\,dx + (1 + xe^y)\,dy = 0$

23. $\dfrac{dx}{dy} + 2x = 6e^y$

24. $2y' + 4y = x^2 - x$

25. $y' - 2y = \cos 3x$

26. $\dfrac{dy}{dx} + y \cot x = 2x + x^2 \cot x$

27. $y' + y = \dfrac{1 + x \log x}{x}$

28. $xy' - y = (x - 1)\,e^x$

Answers

1. $xy = \dfrac{1}{4} x^4 + c$

2. $y(\sec x + \tan x) = \sec x + \tan x - x + c$

3. $y = \sin x + c \cos x$

4. $y(1 + x^2) = \sin x + c$

5. $\dfrac{y}{x+1} = \log (x + 1) + c$

6. $y = (x + 1)^n (e^x + c)$

7. $y = \tan x - 1 + ce^{-\tan x}$

8. $y = \tan^{-1} x - 1 + ce^{-\tan x}$

9. $y(x^2 + 1) = \tan^{-1} x - \dfrac{\pi}{4}$

10. $y = \cos x - 2 \cos^2 x$

11. $x^2 y = \dfrac{x^4}{4} \log x - \dfrac{x^4}{16} + c$

12. $y = 2(\sin x - 1) + ce^{-\sin x}$

13. $y = \dfrac{1}{2} (x^2 - 1) + ce^{-x^2}$

14. $y \sin x = \dfrac{2}{3} \sin^3 x + c$

15. $y = a + c\sqrt{1 - x^2}$

16. $y = \left(1 - \dfrac{1}{x}\right)\left(\dfrac{x^3}{3} + c\right)$

17. $y + 1 + \log x = cx$

18. $y \sin^2 x = x^3 + c$

19. $y = \tan x + c\sqrt{\tan x}$

20. $x = y^3 + cy$

21. $x = \tan^{-1} y - 1 + ce^{-\tan^{-1} y}$

22. $xe^y + y = c$

23. $x = 2e^y + ce^{-2y}$

24. $y = \dfrac{1}{4} (x - 1)^2 + ce^{-2x}$

25. $y = \dfrac{1}{13} (3 \sin 3x - 2 \cos 3x) + ce^{2x}$

26. $y \sin x = x^2 \sin x + c$

27. $y = \log x + ce^{-x}$

28. $y = e^x + cx$